Quantum Physics I exam: Nov 4, 2021.

1] Harmonic oscillator

a) $[a_x^+, a_y^-] = 0$ 5pt.

b) $E = \hbar\omega \cdot (\underline{n_x + n_y + n_z} + \underline{3/2})$

1pt 2pt 2pt

c) 2^{nd} excited state:

$(n_x, n_y, n_z) = (2,0,0), (0,2,0), (0,0,2),$ 1pt

$(0,1,1), (1,0,1), (1,1,0)$ 1pt.

$\Rightarrow$ degeneracy $=$ six → possible values.

3pt

d) $\psi = e^{-iEt/\hbar} \cdot x \cdot y \cdot e^{-\frac{m\omega}{2\hbar}\cdot(x^2+y^2+z^2)}$

3D Gaussian envelope

$E = 7/2 \cdot \hbar\omega$ 1pt.

first excited state for $x \cdot y$ ⇒ 2pt.

2pt.

OR: $x \cdot y \Rightarrow \left(2\frac{m\omega}{\hbar} \cdot x^2 - 1\right)$ 2pt.

(2^{nd} excited state for x).

e)

—— deg = 6 ⇒ when ω_x increases

—— 1 1pt

—— 2 1pt

—— deg = 3 1pt.

2pt

2) Angular momentum

a) $L_-\left(\sin^2\theta \cdot e^{-2i\phi}\right)$

$= -\hbar e^{-i\phi} \cdot \left(\frac{\partial}{\partial\theta} - i \cdot \frac{\cos\theta}{\sin\theta} \cdot \frac{\partial}{\partial\phi}\right)\left(\cdots\right)$

$= -\hbar \cdot e^{-i\phi} \cdot \left(2 \cdot \sin \cdot \cos - i \cdot \frac{\cos}{\sin} \cdot \sin^2 \cdot -2 \cdot i\right) e^{-2i\phi}$

$= 0$ 3pt.

$\Rightarrow$ lowering operator on lowest rung of ladder of spher. harm. Y_2^{-2}.

2pt.

b) $L_+\left(\sin^2\theta \cdot e^{-2i\phi}\right)$

$= +\hbar e^{+i\phi} \cdot \left(\frac{\partial}{\partial\theta} + i \cdot \frac{\cos\theta}{\sin\theta} \cdot \frac{\partial}{\partial\phi}\right)\left(\cdots\right)$

$= +\hbar \cdot e^{+i\phi} \cdot \left(2 \cdot \sin \cdot \cos + \text{same}\right) \cdot e^{-2i\phi}$

$= 4\hbar \cdot e^{-i\phi} \cdot \sin\theta \cdot \cos\theta$

3pt.

$\Rightarrow$ raising operator on Y_2^{-2} gives $\sim Y_2^{-1}$.

2pt.

c) $L_- L_+\left(\sin^2\theta \cdot e^{-2i\phi}\right)$

$= -\hbar e^{-i\phi} \cdot \left(\frac{\partial}{\partial\theta} - i \cdot \frac{\cos\theta}{\sin\theta} \cdot \frac{\partial}{\partial\phi}\right)\left(4\hbar \cdot e^{-i\phi} \cdot \sin\theta \cdot \cos\theta\right)$

$= -4\hbar^2 e^{-i\phi} \cdot \left\{e^{-i\phi}\left(\cos^2 - \sin^2\right) - \cos^2 \cdot e^{-i\phi}\right\}$

$= +4\hbar^2 \cdot e^{-2i\phi} \cdot \sin^2(\theta)$

$\Rightarrow \left(L_+L_- - L_-L_+\right)\left(\sin^2\theta \, e^{-2i\phi}\right)$

$= -4 \cdot \hbar^2 \cdot \left(\sin^2\theta \cdot e^{-2i\phi}\right)$

3pt.

$[L_+, L_-] = 2\hbar \cdot L_z$

and indeed $L_z Y_2^{-2} = -2\hbar \cdot Y_2^{-2}$

so this agrees with the explicit calculation. 2pt.

d) cannot be an eigenstate, 2pt.

as $\sigma_{L_x} \cdot \sigma_{L_y}$ is bounded from below

by $\langle L_z \rangle \neq 0$. $\Rightarrow$ $\sigma_{L_x} \neq 0$, 3pt.

$\sigma_{L_y} \neq 0$.

3) <u>Delta-functions</u>.

a) $\psi = \quad A \cdot e^{+ikx} + B \cdot e^{-ikx} \quad | \quad C \cdot e^{ikx} + D \cdot e^{-ikx} \quad | \quad E \cdot e^{ikx}$ 5pt.

b) $A + B = C + D = E$ 1pt.

$$A \cdot \left(1 + \frac{2mi}{\hbar^2 k} \cdot \alpha\right) - B\left(1 - \text{—}\right) = C - D$$

$$C \cdot \left(1 + \frac{2mi}{\hbar^2 k} \cdot \beta\right) - D\left(1 - \ldots\right) = E$$ 2pt.

five variables $(A, \ldots, E)$

four conditions. Solving linear system

$$\Rightarrow \quad B = \frac{mi(\alpha+\beta)}{\hbar^2 k - mi(\alpha+\beta)} \cdot A, \quad E = \frac{\hbar^2 k}{(\ldots)} \cdot A.$$

$$R = \frac{|B|^2}{|A|^2} = \frac{m^2(\alpha+\beta)^2}{\hbar^4 k^2 + m^2(\alpha+\beta)^2} \qquad T = \frac{|E|^2}{|A|^2} = \frac{\hbar^4 k^2}{\ldots\ldots}$$

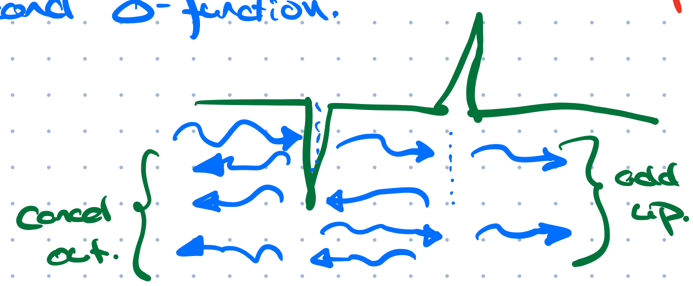
2pt.

c) transparent for $\alpha + \beta = 0$ 1pt.

destructive interference from

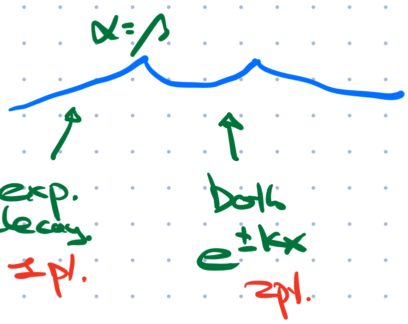
2pt contributions that bounce back from

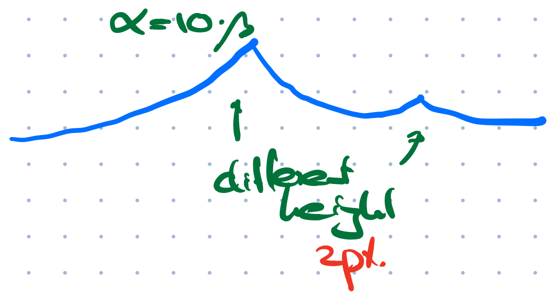
first δ-function & that bounce back from second δ-function. 2pt.



d) No[2pt] - as the two contributions do not differ by an integer number times the wavelength, they are not perfectly in- / out of phase. ⇒ no perfect destructive interference. 3pt.

e)





4] Entanglement.

a) $\psi = a \cdot |10\rangle + b \cdot |01\rangle$ 3pt

with $|a|^2 + |b|^2 = 1$ 2pt.

b) yes, 2pt e.g. $\frac{1}{\sqrt{2}} \left(|10\rangle + |01\rangle \right)$ 3pt

c) 2pt $|10\rangle$ or $|01\rangle$ 3pt

d)

2pt yes, a local measurement collapses the wavefunction everywhere (also other side of the Universe). 3pt.